Solution key of Homework # 5

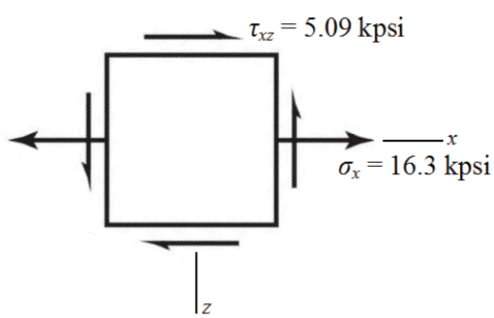
Question #1

(a) Rod *AB* experiences constant torsion throughout its length, and maximum bending moment at the wall. Both torsional shear stress and bending stress will be maximum on the outer surface. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be at the wall, at either the top (compression) or the bottom (tension) on the *y* axis. We will select the bottom element for this analysis.

(b) Transverse shear is zero at the critical stress elements on the top and bottom surfaces.

$$\sigma_x = \frac{Mc}{I} = \frac{M(d/2)}{\pi d^4/64} = \frac{32M}{\pi d^3} = \frac{32(8)(200)}{\pi (1)^3} = 16\,297 \text{ psi} = 16.3 \text{ kpsi}$$

$$\tau_{xz} = \frac{Tr}{J} = \frac{T(d/2)}{\pi d^4/32} = \frac{16T}{\pi d^3} = \frac{16(5)(200)}{\pi (1)^3} = 5093 \text{ psi} = 5.09 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xz})^2} = \frac{16.3}{2} \pm \sqrt{\left(\frac{16.3}{2}\right)^2 + (5.09)^2}$$

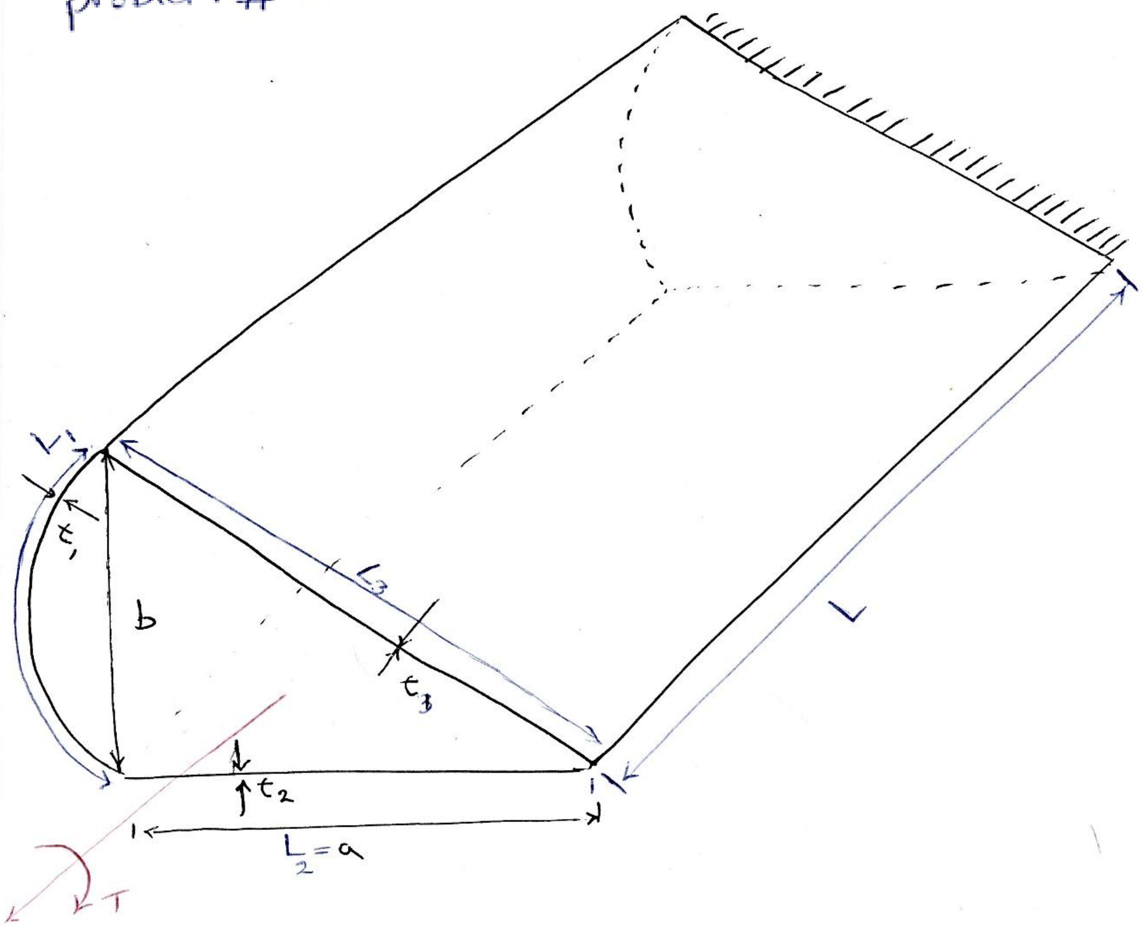
$$\sigma_1 = 17.8 \text{ kpsi} \qquad Ans.$$

$$\sigma_2 = -1.46 \text{ kpsi} \qquad Ans.$$

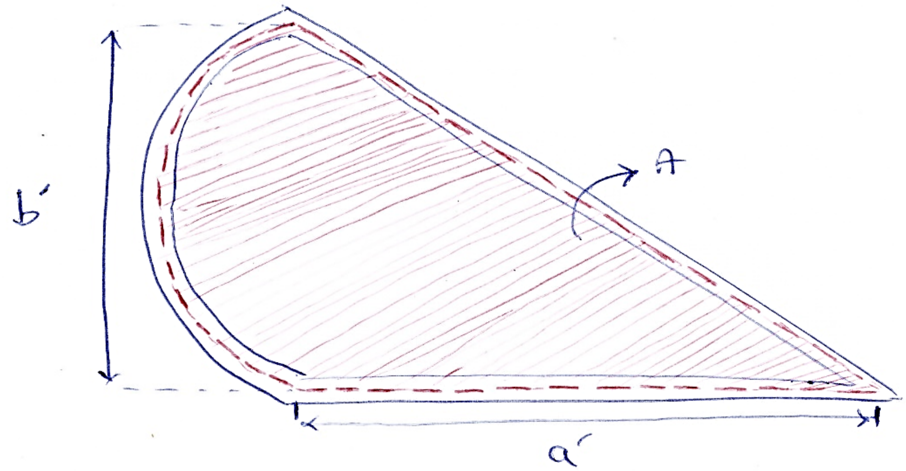
$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xz})^2} = \sqrt{\left(\frac{16.3}{2}\right)^2 + (5.09)^2} = 9.61 \text{ kpsi} \qquad Ans.$$

Homework # 5

problem # 2

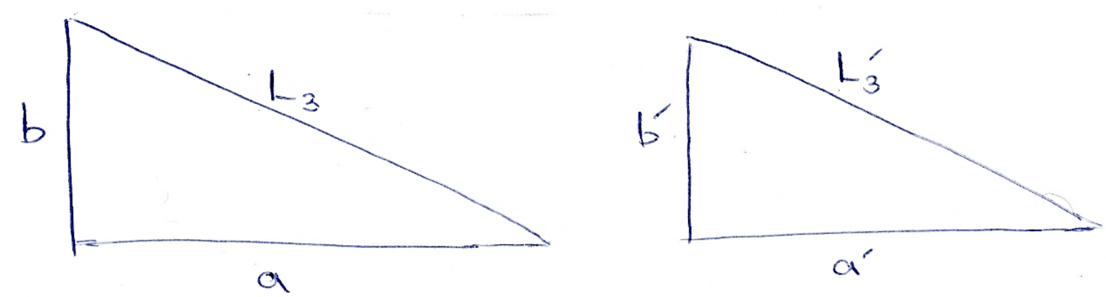


$$\theta = \frac{TL}{4GA^2} \oint \frac{ds}{t} \Rightarrow \frac{\theta}{L} = \frac{T}{4GA^2} \oint \frac{ds}{t}$$



$$b' = b - 2\left(\frac{t_1}{2}\right) = 4 - 2 \times \frac{5}{2} \times 10^{-3} = 3.995 \text{ m}$$

to find the Length of dash-Line triangle, we employ the proportions properties of triangles;



$$\frac{a'}{a} = \frac{b'}{b} \Rightarrow a' = \frac{a}{b} b' = \frac{5}{4} \times 3.99 = 4.99375 \text{ m}$$

$$L_3 = \sqrt{a^2 + b^2} = \sqrt{41} = 6.403$$

the area enclosed by dash-Line is ;

$$A = \frac{\pi b'^2}{4} + \frac{1}{2} b' a' = \frac{\pi}{4} \times (3.995)^2 + \frac{1}{2} (3.995 \times 4.99375)$$

$$\boxed{A = 16.243 \ m^2}$$

$$\oint \frac{ds}{t} = \sum \frac{L_i}{t_i} = \frac{6.403}{0.01} + \frac{5}{0.01} + \frac{\pi (4)/2}{0.005} = 2396.95$$

$$\Rightarrow T = 4G A^2 . \left(\frac{\theta}{L}\right) \times \frac{1}{\oint \frac{ds}{t}} = 4 \times (27 \times 10^9) \times \left(5 \times \frac{\pi}{180}\right) \times \frac{1}{2396.95}$$

$$\Rightarrow \boxed{T = 1.04 \times 10^9 \ Nm}$$